Scattering by an Arbitrary Array of Parallel Wires

JACK H. RICHMOND, SENIOR MEMBER, IEEE

Abstract—Equations are developed for the scattering pattern of an arbitrary array of parallel wires. The wires are assumed to be infinitely long, perfectly conducting, and very small in diameter in comparison with the wavelength. The incident wave is assumed to be TM with respect to the wire axis, but it may have normal or oblique incidence on the wires. The solution includes the interaction effects among all the wires.

The far-field scattering patterns are presented graphically for plane arrays, circular arrays, semicircular arrays, square arrays, and other configurations. If a sufficiently great number of wires is present, it is shown that the scattering pattern approaches that of a solid conducting cylinder of the same cross-section shape as the wire-grid array.

Introduction

ARRAYS OF PARALLEL WIRES are often used instead of plane or curved reflectors to reduce the weight and wind resistance. One type of antenna consists of a grid of parallel wires above a ground plane [1]. If the wires are very long and are closely spaced, these structures can be analyzed successfully in terms of an equivalent shunt susceptance based on the properties of an infinite plane array of parallel wires. The infinite plane array has been analyzed by Wait [2].

An analysis based on infinite plane array data cannot be expected to yield accurate results, however, if the actual array has marked deviations from the planar configuration, if the actual array is only a few wavelengths in width, or if the spacing or wire diameter varies rapidly along the array. The purpose of this paper is to present an accurate solution for the electromagnetic scattering by an arbitrary array of parallel wires in which the number of wires is finite and each wire may have a different diameter. Furthermore, the incident wave need not be a plane wave, but it is assumed that its magnetic field vector is orthogonal to the wire axis. For example, the incident wave might be the field of a line source or an array of line sources at an arbitrary distance from the array of parallel wires.

It is shown that the scattering pattern of the arbitrary array of line sources approaches that of a solid conducting cylinder of the same cross-section shape if a sufficiently large number of wires are present and they are arrayed on a closed curve. Thus, the equations and techniques which are included are useful in obtaining the scattering patterns of circular cylinders, square cylinders, I beams, and so on. With the same equations it is possible to calculate the scattering properties of conducting cylindrical shells such as the parabolic cylinder and the semicircular cylindrical shell. These are of considerable interest both in antenna design and bistatic radar cross-section theory.

An excellent technique has been developed by Andreasen [3] for the analysis of perfectly conducting cylinders of arbitrary cross-section shape. Andreasen's method does not, however, apply to arrays of thin parallel wires such as those considered here. Furthermore, we consider both cylindrical and plane incident waves, whereas earlier papers [3] treat only the plane-wave solution.

The currents I_n induced on the wires are regarded as a set of unknown quantities. If there are N wires, there are N unknown currents. The scattered field of each wire is proportional to the current on the wire, and each individual wire is assumed to have a circular (isotropic) scattering pattern. Thus, a set of N linear equations is obtained for the currents by making the tangential electric field intensity vanish at the center of each wire. The system of linear equations is solved by the method of Crout [4]. After thus determining the complex currents on all the wires, the distant scattering pattern is calculated by considering the array of currents to exist in free space.

The following sections develop the theory and equations for the arbitrary array of parallel wires and present numerical and graphical results for several different arrays.

The Theory of Scattering by Parallel Wires

An arbitrary array of perfectly conducting, parallel, circular wires of infinite length is assumed to exist in unbounded free space as indicated in Fig. 1. Rectangular and cylindrical coordinate systems are selected with the z axis parallel to the axes of the wires, as shown. The incident wave (that is, the field that would exist if the wires were not present) is assumed to be a harmonic TM wave with the following characteristics:

$$H_z^i = 0 \tag{1}$$

and

$$E_z^i = E_t(x, y)e^{-jhz} \tag{2}$$

where h is a constant and the time convention $e^{j\omega t}$ is understood. In general, the incident wave will also have x and y components of electric field intensity, but these components are not involved in the calculations.

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The author is with the Antenna Lab., Dept. of Electrical Engineering, Ohio State University, Columbus, Ohio.

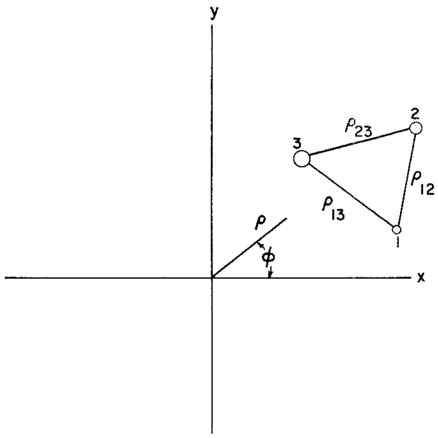

Fig. 1. An array of three parallel wires of different radii, showing the rectangular and cylindrical coordinate systems.

The incident wave may be, for example, one or more plane waves or a continuous spectrum of plane waves traveling in different directions. If a plane wave has an axis of propagation which makes an angle θ_0 with the z axis, its field is given by

$$E_z{}^i = E_i(x, y)e^{-jkz \cos \theta_0} \sin \theta_0 \tag{3}$$

where

$$E_i(x, y) = Ae^{-jk(x \sin \theta_0 \cos \phi_0 + y \sin \theta_0 \sin \phi_0)} \tag{4}$$

and

$$k^2 = \omega^2 \mu \epsilon. \tag{5}$$

The angle θ_0 and ϕ_0 are the angular coordinates of the axis of propagation in the spherical coordinate system.

Another interesting example is an incident field consisting of one or more cylindrical waves of the following form:

$$E_z{}^i = E_i(x, y)e^{-jhz} \tag{6}$$

where

$$E_i(x, y) = AH_n{}^{(2)}(g\rho) \cos n\phi_0 \tag{7}$$

and

$$g^2 + h^2 = k^2. \tag{8}$$

The symbol $H_n{}^{(2)}(g\rho)$ represents the Hankel function of the second kind of order n.

Actually, the only data needed on the incident field are its z axis phase constant h, its frequency or its phase constant k, and the z component of its electric field intensity evaluated at the center of each wire $E_i(x_n, y_n)$.

The current density induced on the surface of wire n will have only a z component, which can be expressed in a Fourier series as follows:

$$J_n(a, \phi, z) = \sum_{i=0}^{\infty} (a_i \cos i\phi + b_i \sin i\phi)e^{-jhz}. \tag{9}$$

If the wire radius is very small in comparison with the wavelength, and if the field incident on the wire has no strong variations over the cross section of the wire, the foregoing series for the current density will converge rapidly. Now consider the scattered field of wire n, evaluated at an observation point at a distance ρ_n from the axis of wire n. This scattered field can be thought of as the field generated by the surface-current density J_n in (9), radiating in unbounded free space. If the observation distance ρ_n is much greater than the wire radius, the scattered field of wire n will be essentially just the field radiated by the zero-order or uniform component of the current density. In other words, the zero-order mode dominates over the higher-order modes except in the near vicinity of the wire. This can be demonstrated analytically by a boundary-value solution, but a heuristic explanation will suffice here. Obviously the net current I_n on the wire is determined by the zero-order mode alone.

It can readily be shown that the field of a harmonic current I_n uniformly distributed on a circular cylinder of radius a_n has a z component given by

$$E_n = -(\omega\mu g^2/4k^2)I_nJ_0(ga_n)H_0{}^{(2)}(g\rho_n)e^{-jhz} \quad (\text{for } \rho_n \geq a_n) \tag{10}$$

where $J_0(ga_n)$ represents the Bessel function of zero order, and g is given by (8). It is convenient to define a "modified current" I_n' as follows:

$$I_n' = (\omega\mu g^2/4k^2)I_nJ_0(ga_n). \tag{11}$$

In terms of the modified current, the scattered field of wire n is given by

$$E_n = -I_n'H_0{}^{(2)}(g\rho_n)e^{-jhz} \quad (\text{for } \rho_n \geq a_n). \tag{12}$$

Since the wires are assumed to be perfectly conducting, the z component of the total electric field intensity (incident field plus scattered field) must vanish at the surface of each wire and everywhere within each wire. If this condition is enforced at the center of wire m, the following result is obtained:

$$\sum_{n=1}^{N} H_0{}^{(2)}(g\rho_{mn})I_n' = E_i(x_m, y_m) \tag{13}$$

where ρ_{mn} is the distance between wire m and wire n,

$$\rho_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2} \tag{14}$$

and

$$\rho_{mm} = a_m. \tag{15}$$

Equations (13), (14), and (15) are based on the assumption that the scattered field of each wire is the same at the center and at the surface of the wire. This is a reasonable approximation for wires with radii much smaller than the wavelength.

As m takes on the values 1, 2, 3, $\cdots$, N, (13) yields a set of N linear equations for the N unknown currents I_n'. These linear equations (with complex coefficients) are solved with the aid of a digital computer. The method of Crout [4] has been found to be advantageous for this purpose.

Once the currents have been determined in this manner, the scattered field can be calculated as follows:

$$E_z^s = -\sum_{n=1}^{N} I_n' H_0^{(2)}(g\rho_n) e^{-jhz}. \tag{16}$$

Equation (16) is obtained by summing on (12).

We have made two assumptions which limit the range of validity of the solution. First, the scattered field of a given wire is assumed to be the same at the center of that wire as it is at its surface. This is a good approximation if ka is less than 0.2 (i.e., $a \leq 0.03\lambda$) in which case the field at the center differs by less than 1 per cent from the field at the surface. Second, we assume that each wire has a circular scattering pattern when we calculate its scattered field at the center of another wire. By considering the exact solution for plane-wave scattering by a single wire, it is easy to show that this is a good approximation when ka is less than 0.2 if the wires are not too close together. For example, the distance between the centers of adjacent wires should be at least 6 radii if $ka = 0.2$. In fact, a minimum spacing of 6 radii is suggested even when ka is less than 0.2.

The Distant Scattering Pattern

If the observation point is a great distance from the wires, (16) can be simplified by means of the asymptotic formula for the Hankel function,

$$H_0^{(2)}(x) = \sqrt{2j/\pi x}\, e^{-jx} \qquad (\text{if } x \gg 1). \tag{17}$$

A further simplification is achieved by noting that the distance from wire n (located at x_n, y_n) to the observation point (at ρ, ϕ) is given by

$$\rho_n = \rho - x_n \cos \phi - y_n \sin \phi. \tag{18}$$

From (16), (17), and (18), the distant scattered field is given by

$$E_z^s = -\sqrt{2j/\pi g\rho}\, e^{-jg\rho} e^{-jhz} \sum_{n=1}^{N} I_n' \cdot e^{jg(x_n \cos\phi + y_n \sin\phi)}. \tag{19}$$

The "distant scattering pattern" $E(\phi)$ is defined by the summation in (19),

$$E(\phi) = \sum_{n=1}^{N} I_n' e^{jg(x_n \cos\phi + y_n \sin\phi)}. \tag{20}$$

Numerical Results

Using the equations previously given and an IBM 1620 digital computer, numerical calculations have been carried out for plane arrays of parallel wires, circular arrays, square arrays, and I beam arrays. In this section some representative results are presented.

Consider first a plane array of wires with uniform spacing and radii. The incident wave is a plane wave whose axis of propagation is perpendicular to the axes of the wires. Figure 2 shows the distant scattering pattern of this plane array of wires and the numbering system for the wires. It may be observed in Fig. 2 that the solutions satisfy the reciprocity theorem; that is, the scattering pattern has exactly the same magnitude (and phase) when $\phi_0 = 0$ and $\phi = 40°$ as it has when $\phi_0 = 40°$ and $\phi = 0$.

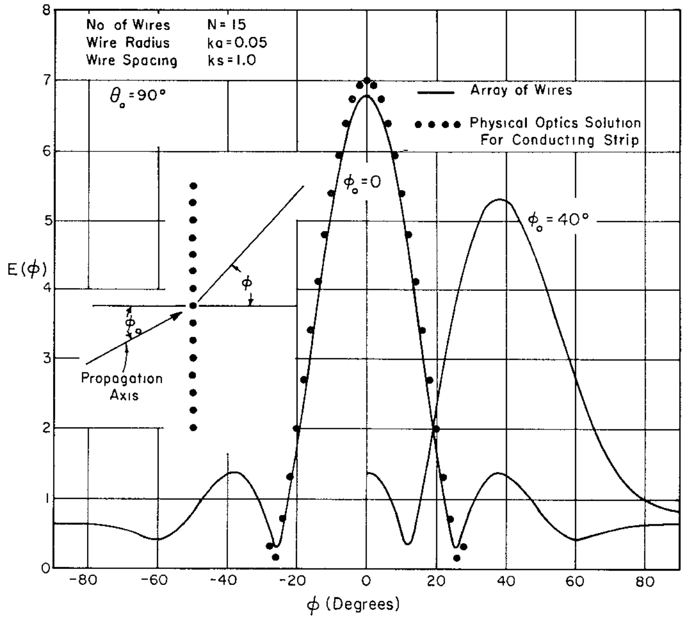

Fig. 2. Scattering pattern of a plane array of 15 parallel wires for a plane wave incident at angles of 0° and 40°, and the physical-optics solution for a plane conducting strip of the same width.

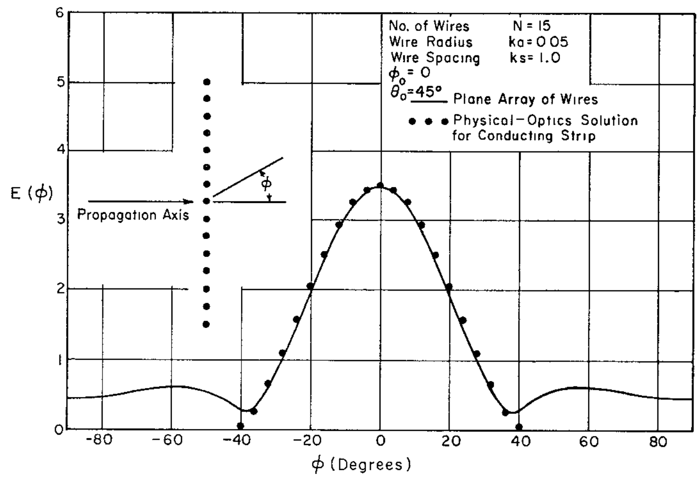

Fig. 3. Scattering pattern of a plane array of 15 parallel wires, and the physical-optics solution for a plane conducting strip of the same width.

The physical-optics solution for plane-wave scattering by a thin metal strip of infinite height and width L is also shown in Fig. 2 for comparison. Normal incidence is assumed (i.e., $\phi_0 = 0$), and the width L is taken equal to the width of the plane array of wires ($kL = 14$). The surface current density on the metal strip is assumed to be zero on the shadow side and $J = 2\hat{n} - H^i$ on the illumi-

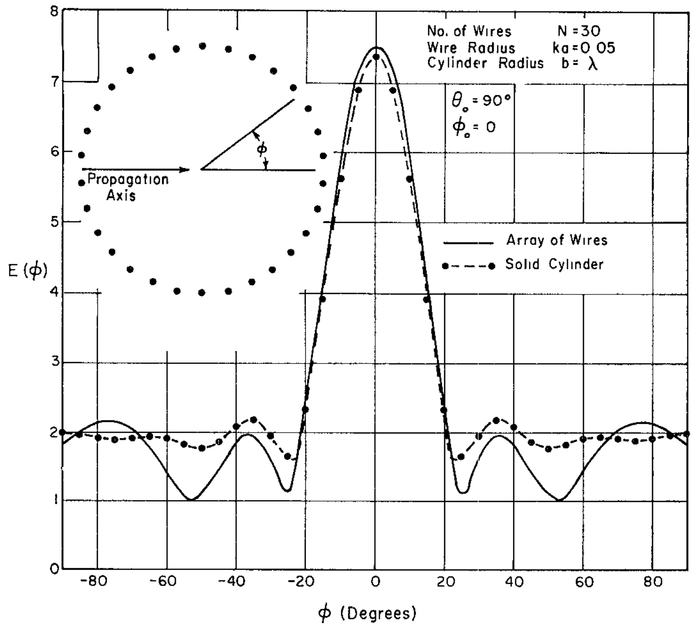

Fig. 4. Scattering pattern of a circular array of 30 parallel wires, and the exact solution for a solid conducting cylinder of the same radius.

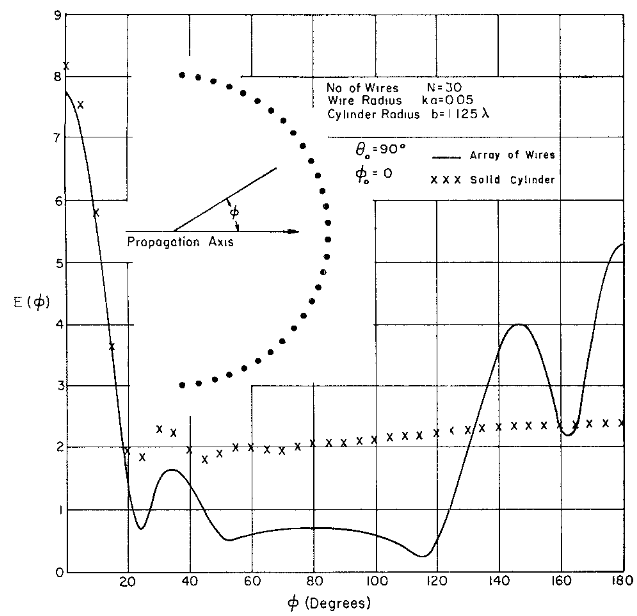

Fig. 5. Scattering pattern of a semicircular array of 30 parallel wires, and the exact solution for a solid conducting cylinder of the same radius.

nated side, where $\hat{n}$ is the unit outward normal on the conducting surface. Thus, the physical-optics solution for the scattering pattern of the thin metal strip for $\phi_0 = 0$ is given by

$$E(\phi) = E_0 \frac{\sin [(kL/2) \sin \theta_0 \sin \phi]}{\sin \phi} \sin \theta_0 \qquad (21)$$

where E_0 represents the total electric field intensity of the incident plane wave (not just the z component). It may be observed in Fig. 2 that the scattering pattern of

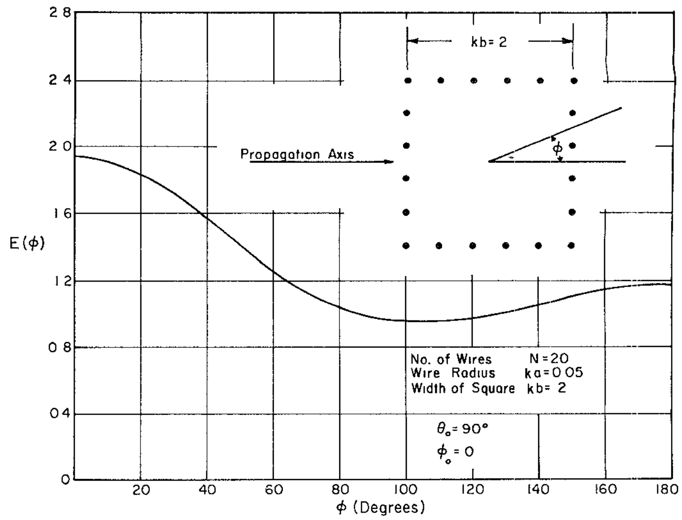

Fig. 6. Scattering pattern of an I beam array of 15 wires for normal incidence.

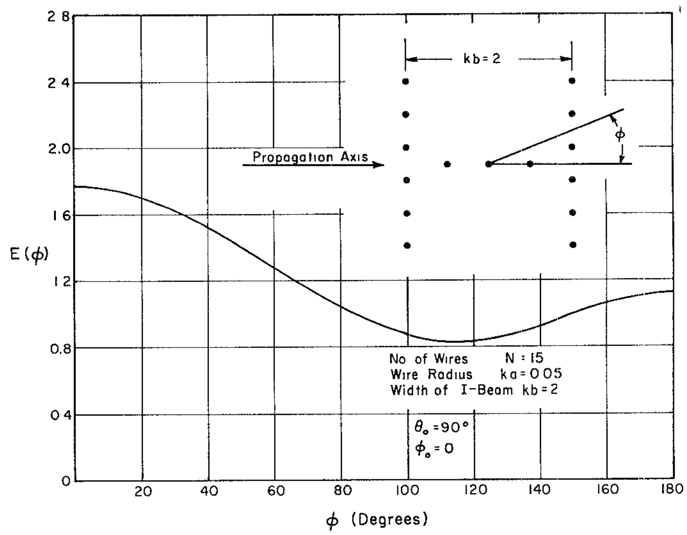

Fig. 7. Scattering pattern of a square array of 20 wires for normal incidence.

the plane array of wires agrees closely with that of a thin metal strip of the same width. It has been found that a solid conducting cylinder or a hollow conducting cylindrical shell of arbitrary cross section can be closely approximated by an array of parallel wires if a sufficiently large number of wires is employed. A "good" approximation is obtained for the scattering pattern if five wires are used per wavelength along the periphery of the conducting cylinder, and an "excellent" approximation will result if ten wires are used per wavelength.

Figure 3 shows the scattering pattern of the plane array of wires for a plane wave with a propagation direction defined by the angles $\theta_0 = 45°$ and $\phi_0 = 0$. The incident plane wave is assumed to have an electric field intensity of 1 V/m; its z component is thus 0.70711 V/m. The physical-optics solution (21) is shown in Fig. 3 for comparison.

Now consider a circular array of 30 wires with uniform spacing and radii. The incident wave is a plane wave whose axis of propagation is perpendicular to the axes of the wires. Figure 4 shows the scattering pattern of the circular array of 30 wires. For comparison, the exact scattering pattern of a conducting circular cylinder of the same radius is also shown. It may be observed that the scattering properties of the wire-grid cylinder closely approximate those of the solid conducting cylinder if the spacing between adjacent wires is sufficiently small. In this example there are approximately 5 wires per wavelength around the circumference of the cylinder. By using a larger number of wires per wavelength it is possible to simulate the solid conducting cylinder with greater accuracy.

Figure 5 shows the results for a semicircular array of 30 wires. The scattering pattern of a solid conducting cylinder of the same radius is also shown in Fig. 5 for comparison. It is seen that the semicircular array has nearly the same forward scattering as the complete circular cylinder, but the backscattering is considerably increased.

Figure 6 shows the scattering pattern for a square array of 20 wires. The scattering pattern agrees closely with experimental measurements and with the results of Mei and Van Bladel [5] for a perfectly conducting cylinder of square cross section.

Figure 7 shows the results for an array of 15 wires on the cross section of an I beam. The previous results lead us to believe that the scattering properties of this array of wires closely approximate these of a solid conducting I beam of the same dimensions.

Conclusions

Equations are given for the scattering pattern of an arbitrary array of parallel wires. The incident wave is assumed to have no magnetic field component parallel with the axes of the wires. The solution provides numerical data on the complex currents induced on the wires and the distant scattering pattern of the array of wires. The effects of interaction among the wires are taken into account automatically.

Numerical results are included for the following arrays: plane, circular, semicircular, square, and I beam. It is shown that the scattering properties of a solid conducting cylinder of arbitrary cross section, or a thin conducting cylindrical shell, can be simulated accurately by an appropriate array of wires. To accomplish this, there must be at least five wires per wavelength along the periphery of the cylinder or cylindrical shell.

If the wires are closely spaced, the Lagrange interpolation formulas may be employed to permit a substantial increase in the maximum number of wires.

The solution for an unsymmetrical array of 15 wires requires about 25 minutes with the IBM 1620 computer to obtain the complete scattering pattern with increments of 5° in the scattering angle. With an IBM 7094 digital computer, the solution for an array of 50 wires is obtained in one minute.

A few of the many possible applications of the technique are in studying diffraction by a slit in a ground plane, and the patterns of a two-dimensional horn antenna excited by a line source near the vertex. The method has been employed in studies of a vertical antenna near a metal smokestack (a Navy problem) and a parabolic antenna with a line source located away from the focal axis.

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